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Crossing-free straight-line drawing of graphs on the flat torus*

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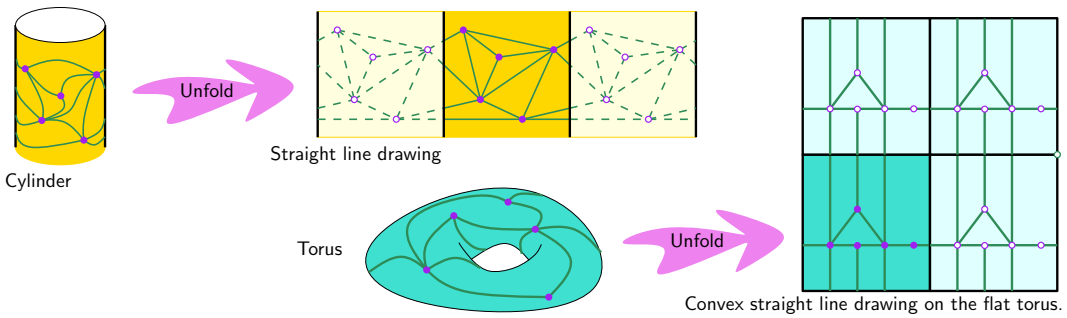

1 Introduction

The problem of efficiently computing straight-line drawings of planar graphs has attracted a lot of attention in the past. In this paper we took interest on *straight line drawings* for graphs drawn on the cylinder or on the torus. More precisely, we look at straight line drawing of unfolded periodic representations of the cylinder and the torus (see Figure).

2 Main Result

Theorem 1 *For each essentially* 3*-connected*[1] *toroidal map* G*, one can compute in linear time a weakly convex crossing-free straight-line drawing of* G *on a periodic regular grid* $\mathbb{Z}/w\mathbb{Z} \times \mathbb{Z}/h\mathbb{Z}$*, where —with* n *the number of vertices and* c *the number of edges of the shortest non contractible loop—* $w \leq 2n$ *and* $h \leq 1 + 2n(c+1)$*. Since* $c \leq \sqrt{2n}$*, the grid area is* $O(n^{5/2})$*.*

3 Sketch of Algorithm

We extend the incremental straight-line drawing algorithm of de Fraysseix, Pach and Pollack [2] (in the triangulated case) and of Kant [3] (in the 3-connected case).

The case of triangulations on a cylinder with the following properties: — no chords on the boundary, — no loops, — and no 2-cycles can be solved generalizing de Fraysseix *et al.* algorithm [2] . It needs to find a good *shelling* order to add the vertices and draw incrementally the triangulation preserving a good shape of its boundary. Allowing chords, loops, and 2-cycles is possible by a careful subdivision of the triangulation in pieces and gluing the result of the simple case. Generalizing to the torus can be done by cutting the graph along a short non contractible cycle. Adaptation from triangulation to maps is similar to the planar case [3].

References

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[2] H. de Fraysseix, J. Pach and R. Pollack. How to draw a planar graph on a grid. *Combinatorica*, 10(1):41–51, 1990. doi:10.1007/BF02122694.

[3] G. Kant. Drawing planar graphs using the canonical ordering. *Algorithmica*, 16(1):4–32, 1996. doi:10.1007/BF02086606.

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[1] 3-connected in the periodic drawing,

This is a short abstract of a presentation given at the Workshop on Geometric Structures with Symmetry and Periodicity, part of the CG Week 2014. It has been made public for the benefit of the community and should be considered a preprint rather than a formally reviewed paper. Thus, this work may appear in a conference with formal proceedings and/or in a journal.